INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 4.11]

NORMAL DISTRIBUTION

Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. [Maximum mark: 7]

The random variable X is normally distributed with $\mu = 100$ and $\sigma = 20$, i.e. $N(100, 20^2)$.

(a) Find the probabilities

(i) $P(X < 90)$

(ii) $P(90 < X < 130)$

(iii) $P(X > 130)$ [3]

(b) Sketch a diagram to represent the information in question (a). [2]

(c) 90 is m standard deviations **below** the mean and 130 is n standard deviations **above** the mean. Write down the values of m and n. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

2. [Maximum mark: 7]

The random variable X follows a normal distribution with $\mu = 100$ and $\sigma = 20$.

(a) Given that $P(X < a) = 0.8$ find the value of a. [2]

(b) Given that $P(X > b) = 0.3$ find the value of b. [2]

(c) Find Q_1 and Q_3. [3]

.....

.....

.....

.....

.....

.....

.....

.....

3. [Maximum mark: 6]

The random variable X is normally distributed with $\mu = 100$. It given that $P(X > 130) = 0.2$

Write down the values of the following probabilities

(a) $P(X < 130)$ [1]

(b) $P(X < 70)$ [2]

(c) $P(100 < X < 130)$ [1]

(d) $P(70 < X < 130)$ [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

4. [Maximum mark: 9]

The mass of packets of a breakfast cereal is normally distributed with a mean of 750 g and standard deviation of 25 g.

(a) Find the probability that a packet chosen at random has mass

(i) less than 740 g; (ii) at least 780 g; (iii) between 740 g and 780 g. [5]

(b) Two packets are chosen at random. What is the probability that both packets have a mass which is less than 740 g? [2]

(c) The mass of 70% of the packets is more than x grams. Find the value of x. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

5. [Maximum mark: 4]

The heights, H, of the people in a certain town are normally distributed with mean 170 cm and standard deviation 20 cm.

(a) A person is selected at random. Find the probability that his height is less than 185 cm. [2]

(b) Given that $\mathrm{P}(H > d) = 0.6808$, find the value of d. [2]

.....

.....

.....

.....

.....

6. [Maximum mark: 4]

The heights of a group of students are normally distributed with a mean of 160 cm and a standard deviation of 20 cm.

(a) A student is chosen at random. Find the probability that the student's height is greater than 180 cm. [2]

(b) In this group of students, 11.9% have heights less than d cm. Find the value of d. [2]

.....

.....

.....

.....

.....

7. [Maximum mark: 5]

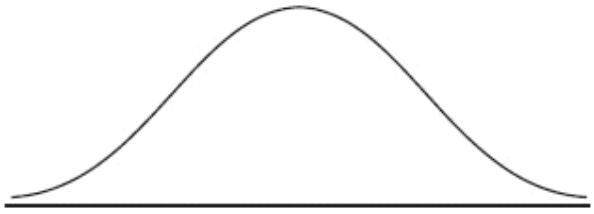
The weights of a group of children are normally distributed with a mean of 22.5 kg and a standard deviation of 2.2 kg.

(a) Write down the probability that a child selected at random has a weight more than 25.8 kg. [2]

(b) Of the group 95% weigh less than k kilograms. Find the value of k. [2]

(c) The diagram below shows a normal curve. On the diagram, shade the region that represents the following information:

87% of the children weigh less than 25 kg [1]



.....

.....

.....

.....

.....

.....

8. [Maximum mark: 5]

Intelligence Quotient (IQ) in a certain population is normally distributed with a mean of 100 and a standard deviation of 15.

(a) What percentage of the population has an IQ between 90 and 125? [2]

(b) If two persons are chosen at random from the population, what is the probability that both have an IQ greater than 125? [3]

.....

.....

.....

.....

.....

.....

9. [Maximum mark: 4]

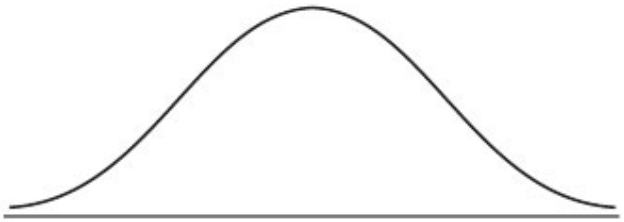
The heights of certain plants are normally distributed. The plants are classified into three categories.

The shortest 12.92% are in category A.

The tallest 10.38% are in category C.

All the other plants are in category B with heights between r cm and t cm.

(a) Complete the following diagram to represent this information.



[2]

(b) Given that the mean height is 6.84 cm and the standard deviation 0.25 cm, find the value of r and of t. [2]

.....

.....

.....

.....

.....

10. [Maximum mark: 5]

It is claimed that the masses of a population of lions are normally distributed with a mean mass of 310 kg and a standard deviation of 30 kg.

(a) Calculate the probability that a lion selected at random will have a mass of 350 kg or more. [2]

(b) The probability that the mass of a lion lies between a and b is 0.95, where a and b are symmetric about the mean. Find the value of a and of b. [3]

.....

.....

.....

.....

.....

.....

.....

11. [Maximum mark: 7]

Residents of a small town have savings which are normally distributed with a mean of $ 3000 and a standard deviation of $500.

(a) What percentage of townspeople have savings greater than $ 3200? [2]

(b) Two townspeople are chosen at random. What is the probability that **both** of them have savings between $ 2300 and $ 3300? [3]

(c) The percentage of townspeople with savings less than d dollars is 74.22%. Find d. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

12. [Maximum mark: 7]

A company manufactures television sets. They claim that the lifetime of a set is normally distributed with a mean of 80 months and standard deviation of 8 months.

(a) What proportion of television sets break down in less than 72 months? [2]

(b) (i) Calculate the proportion of sets with a lifetime between 72 and 90 months.

(ii) Illustrate this proportion by a sketch of a normal distribution curve. [3]

(c) If a set breaks down in less than x months, the company replace it free of charge. They replace 4% of the sets. Find the value of x. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

13. [Maximum mark: 4]

A random variable X is distributed normally with a mean of 100 and a variance of 100.

(a) Find the value of X that is 1.12 standard deviations **above** the mean. [2]

(b) Find the value of X that is 1.12 standard deviations **below** the mean. [2]

.....

.....

.....

.....

.....

14. [Maximum mark: 9]

The heights of trees in a forest are normally distributed with mean height 17 metres. One tree is selected at random. The probability that a selected tree has a height greater than 24 metres is 0.06.

(a) Find the probability that the tree selected has a height less than 24 metres. [2]

(b) The probability that the tree has a height less than D metres is 0.06. Find D. [3]

(c) A woodcutter randomly selects 200 trees. Find the expected number of trees whose height lies between 17 metres and 24 metres. [4]

.....

.....

.....

.....

.....

.....

.....

.....

15. [Maximum mark: 5]

In a country called *Tallopia,* the height of adults is normally distributed with a mean of 187.5 cm and a standard deviation of 9.5 cm.

(a) What percentage of adults in *Tallopia* have a height greater than 197 cm? [2]

(b) A standard doorway in *Tallopia* is designed so that 99 % of adults have a space of at least 17 cm over their heads when going through a doorway. Find the height of a standard doorway in *Tallopia*. Give your answer to the nearest cm. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

16. [Maximum mark: 7]

The random variable X follows a normal distribution with $\mu = 100$ and $\sigma = 5$.

(a) Write down the value of $P(X = 102)$. [1]

(b) Find the probability that $X = 102$ if the value of X is rounded to the nearest whole number. [2]

(c) Find the conditional probabilities

(i) $P(X > 102 \mid X > 100)$ (ii) $P(X < 102 \mid X > 100)$ [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

17. [Maximum mark: 6]

The lengths of a particular species of lizard are normally distributed with a mean length of 50 cm and a standard deviation of 4 cm, A lizard is chosen at random.

(a) Find the probability that its length is greater than 45 cm. [2]

(b) Given that its length is greater than 45 cm, find the probability that its length is greater than 55 cm. [4]

.....

.....

.....

.....

.....

.....

.....

18. [Maximum mark: 10]

The weight of the population of a particular animal is normally distributed with mean 300kg and standard deviation 40kg. Animals above 350kg are considered **overweight**.

(a) We select 1000 animals. Find the **expected number** of the overweight animals. [2]

(b) We select **two** animals. Find the probability that

(i) both animals are overweight.

(ii) only one of the animals is overweight.

(iii) at least one of the animals is overweight. [4]

(c) We select **eight** animals. Find the probability that

(i) exactly half of the animals are overweight.

(ii) at least one of the animals is overweight. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

B. Paper 2 questions (LONG)

19. [Maximum mark: 11]

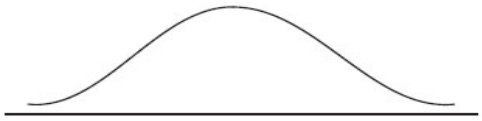
A box contains a large number of biscuits. The weights of biscuits are normally distributed with mean 7 g and standard deviation 0.5 g.

(a) One biscuit is chosen at random from the box. Find the probability that this biscuit

(i) weighs less than 8 g; (ii) weighs between 6 g and 8 g. [4]

(b) Five percent of the biscuits in the box weigh less than *d* grams.

(i) Complete the following normal distribution diagram, to represent this information, by indicating d, and shading the appropriate region.



(ii) Find the value of d. [5]

(c) Find the interquartile range of wights of the biscuits. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

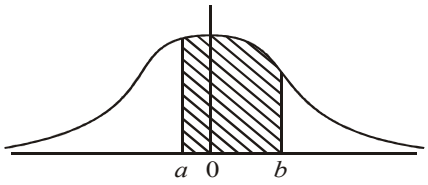
.....

.....

20. [Maximum mark: 10]

The lifespan of a particular species of insect is normally distributed with a mean of 57 hours and a standard deviation of 4.4 hours.

(a) The probability that the lifespan of an insect of this species lies between 55 and 60 hours is represented by the shaded area in the following diagram. This diagram represents the standard normal curve.



(i) Write down the values of a and b. [2]

(ii) Find the probability that the lifespan of an insect of this species is

(a) more than 55 hours; (b) between 55 and 60 hours. [3]

(b) 90% of the insects die after t hours.

(i) Represent this information on a standard normal curve diagram, similar to the one given in part (a), indicating clearly the area representing 90%. [2]

(ii) Find the value of t. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

21. [Maximum mark: 12]

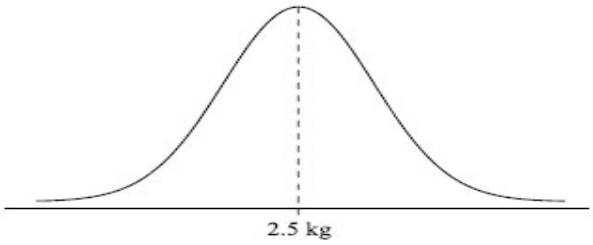
The weights of chickens for sale in a shop are normally distributed with mean 2.5 kg and standard deviation 0.3 kg.

(a) A chicken is chosen at random.

(i) Find the probability that it weighs less than 2 kg.

(ii) Find the probability that it weighs more than 2.8 kg.

(iii) In the diagram below, shade the areas that represent the probabilities from parts (i) and (ii).



(iv) Find the probability that it weighs between 2 kg and 2.8 kg (4sf). [7]

(b) A customer buys 10 chickens.

(i) Find the probability that all 10 chickens weigh between 2 kg and 2.8 kg.

(ii) Find the probability that at least 7 of the chickens weigh between 2 kg and 2.8 kg. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

22. [Maximum mark: 13]

In a large school, the heights of all fourteen-year-old students are measured.

The heights of the girls are normally distributed with mean 155cm and standard deviation 10cm.

The heights of the boys are normally distributed with mean 160cm and standard deviation 12cm.

(a) Find the probability that a girl is taller than 170 cm. [2]

(b) Given that 10% of the girls are shorter than x cm, find x. [2]

(c) Given that 90% of the boys have heights between q cm and r cm where q and r are symmetrical about 160 cm, and $q < r$, find the value of q and of r. [3]

In the group of fourteen-year-old students, 60% are girls and 40% are boys.

The probability that a girl is taller than 170 cm was found in part (a).

The probability that a boy is taller than 170 cm is 0.202.

A fourteen-year-old student is selected at random.

(d) Calculate the probability that the student is taller than 170 cm. [3]

(e) Given that the student is taller than 170 cm, what is the probability the student is a girl? [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

23. [Maximum mark: 16]

A company buys 44 % of its stock of bolts from manufacturer A and the rest from manufacturer B. The diameters of the bolts produced by each manufacturer follow a normal distribution with a standard deviation of 0.16 mm.

The mean diameter of the bolts produced by manufacturer A is 1.56 mm.

The mean diameter of the bolts produced by manufacturer B is 1.63 mm.

(a) Find the percentage of the bolts produced by manufacturer B which have a diameter less than 1.52. [3]

A bolt is chosen at random from the company's stock.

(b) Show that the probability that the diameter is less than 1.52 mm is 0.312, to three significant figures. [4]

(c) The diameter of the bolt is found to be less than 1.52 mm. Find the probability that the bolt was produced by manufacturer B. [3]

(d) Manufacturer B makes 8000 bolts in one day. It makes a profit of $ 1.50 on each bolt sold, on condition that its diameter measures between 1.52 mm and 1.83 mm Bolts whose diameters measure less than 1.52 mm must be discarded at a loss of $0.85 per bolt. Bolts whose diameters measure over 1.83 mm are sold at a reduced profit of $0.50 per bolt.

Find the expected profit for manufacturer B. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

24. [Maximum mark: 14]

Ian and Karl have been chosen to represent their countries in the Olympic discus throw. Assume that the distance thrown by each athlete is normally distributed. According to the records in the past year

Karl's throws have a mean of $59.50\ \text{m}$ and a standard deviation of $3.00\ \text{m}$

Ian's throws have a mean of $60.33\ \text{m}$ and a standard deviation of $1.95\ \text{m}$

(a) Find the percentage of Karl's throws which have been longer than $56\ \text{m}$. [2]

(a) 80 % of Ian's throws have been longer than x metres. Find x, correct to **two** decimal places. [2]

(c) This year, Karl's and Ian's throws still have the same means and standard deviations. In a competition an athlete must have at least one throw of $65\ \text{m}$ or more in the first round to qualify for the final round. Each athlete is allowed three throws in the first round.

(i) Determine which of these two athletes is more likely to qualify for the final on their first throw.

(ii) Find the probability that **both** athletes qualify for the final. [10]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....